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Characterizing Successful Packet Transmission in a Vehicular Ad Hoc Network

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Abstract—**This paper studies the probability of packets being successfully delivered to their intended receivers (vehicles) on a highway by taking multi-user interference, path loss, and fading into account. This probability affects the efficiency of V2V communications for safety purposes in vehicular networks. The presented mathematical framework can be employed in designing the safety applications of VANETs. This has been shown in our simulation results which compare the vehicle collision probability of different models on highways.**

I. INTRODUCTION[1]

During the past decade, the automobile industry has seen a rise in the use of advanced technologies, in order to improve automobile safety. Sadly, however, the fatalities and injuries caused due to automobile accidents have remained at an alarming level. In particular, statistics from 2013 [1] report over five million crashes in the U.S., causing over two million injuries and more than 30,000 fatalities.

With the advancements in Vehicular Ad Hoc Networks (VANET, Fig.1), recent research [2] suggests the use of VANETs to improve the safety on highways. VANETs allow for cross-communication between cars within a close proximity of each other, which can enable them to efficiently and reliably communicate sensitive traffic messages such as crash-relevant information. The Federal Communications Commission has allocated 75 MHz of spectrum in the 5.9 GHz band for Dedicated Short Range Communications (DSRC). To serve as the groundwork for DSRC, the IEEE 802.11p standard was published in the year 2010 for Wireless Access in Vehicular Environments (WAVE) [3].

We need to know about the uncertainty of the packet delivery between two specific vehicles while other vehicles might also transmit simultaneously, thus interfering with the selected packet transmission. Deriving this probability helps us with finding the communication delay to inform each vehicle in a chain while employing vehicular communications. It is desirable to reduce this delay as much as possible by lessening the interference caused by other vehicles. The intensity of the simultaneously active nodes plays a vital role in quantifying the interference experienced by a vehicle in the network.

Our main contributions in this paper are as follows:

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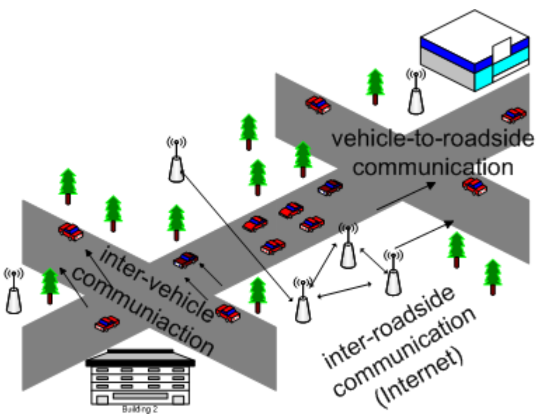

Fig. 1. VANET: Vehicular Ad-hoc NETwork

1) We find the expressions of packet success probability for two specific scenarios: 1. Equal distance between vehicles 2. Vehicles spatially modeled as a Matern hard core point process (Matern HCPP). By both modeling the spatial distributions of the simultaneously active nodes and characterizing the interference, the number of required slots on average for every node to receive safety messages are obtained.
2) We compare the collision probability for the specified models using simulations.

The remainder of this paper is organized as follows. Section II summarizes the related work that has been done in the field of vehicular communications regarding finding the communication parameters suitable for safety applications. We discuss our analysis to obtain packet success probability equations in section III. In section IV, the simulation results are demonstrated. Conclusion is given in section V.

II. BACKGROUND AND LITERATURE REVIEW

[4] simulated two vehicular safety applications and determined the effect of various communication parameters on vehicle crash avoidance through simulations. However, they don't develop any mathematical framework for packet delivery success. [5] proposes a series of repetition-based Media Access

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Control (MAC) protocols to deliver periodic status updates within their useful lifetime to within a specified range. For a scheme in which nodes transmit with a given probability in each slot, [5] derives the Probability of Reception Failure (PRF) at the border of the range of interest. However, they only consider the strongest interferer in their derivation and neglect fading. [13] and [14] discuss the interference modeling for nodes spatially distributed as Poisson Point Processes (PPP). In [11], while the nodes were modeled as PPP, Matern hard core point process type II (MHCPP-II) was employed to model the spatial distribution of access points (APs). The modified Matern CSMA process was presented in [12]. However, the retaining probability was not defined as a function of distances. In addition, their results are not general and are only valid for specific range of intensities of network nodes. The aggregate interference based on the MHCPP-II model for CSMA network was obtained only by simulations in [15]. [16] studied the effect of MAC protocol on the outage probability for CSMA protocols and slotted/unslotted ALOHA. However, the circle capture model was employed in their analysis to obtain the lower-bound on the outage probability.

III. Analysis and Design

Communications between vehicles can help drivers with making proper reactions to the deceleration events especially when a driver cannot either observe or perceive the deceleration of other vehicles due to low visibility, high unexpectedness of the incident, defected brake lights, and many distractions that nowadays exist on the roads. In a network of vehicles, each vehicle transmits with a specific probability in the transmission medium. Large channel access probabilities lead the system to excessive interferences and consequently low probability of packets being successfully received (success probability) while very small values reduces the success probabilities since the probability of the favorite transmission is low itself. [6] states that vehicles in traffic are more likely to follow Poisson distribution under low flow conditions. Under near-capacity conditions, however, the equal distance assumption between vehicles is justified. Therefore, our design is divided into two scenarios: 1. Equal distance model 2. Poisson distribution model. We believe examining these two scenarios gives us a thorough picture of how vehicular communications are affected by different parameters.

Although the Media Access Control (MAC) protocol for DSRC communications is a variation of the conventional CSMA/CA scheme, because of the short length of the packet payload and the broadcast nature of communications, the 4-way handshake anticipated by the standard is not efficient for the dissemination of periodic safety messages. RTS/CTS and ACK message exchanges increase the hidden node problem thus resulting in higher probability of packet collisions [8]. Since the topology of VANETs is highly dynamic, we need protocols which do not need a detailed description of the network topology to schedule packet transmissions. Repetition-based protocols not only reveal this property, but also fight packet collisions due to the problem of hidden nodes. Hence, in this section, we make use of repetition-based protocols for the dissemination of periodic safety messages. A similar approach has been used in other papers, e.g. in [9] and [8].

1) Equal distance:
The MAC scheme that we consider is SSP (Slotted Synchronous P-persistent) where at each slot a node (vehicle) transmits with probability p and receives with probability $1-p$ independent of others. The important assumption is that the slots are synchronized because of the on-board GPS devices. Moreover, since the vehicles are not faced with power constraints, the nodes can increase the transmission power to overcome the interference. In this paper, we consider path loss and Rayleigh fading for formalizing the signal propagation characteristics. If we assume that the nodes transmit with unit power, the received power at distance r is $hr^{-\alpha}$, where $\alpha(>1)$ is the path loss exponent and h is the fading coefficient. Assuming that a node transmits a packet, the probability that a receiver at distance r receives the packet successfully is:

$$\begin{aligned} P_s &= Prob\left(\frac{S}{I} > \beta\right) \\ &= Prob\left(\frac{hr^{-\alpha}}{\sum_{i=-\infty}^{\infty} b_i h_i r_i^{-\alpha}} > \beta\right) \\ &= \frac{\lambda(1+\beta)}{p_{tr}+\lambda(1-p_{tr})+\lambda(1-p_{tr})\beta}\cdot \\ &\prod_{i=-\infty-\{0\}}^{+\infty} \frac{p_i+\lambda(1-p_i)+\lambda(1-p_i)\beta\left(\frac{m}{i}\right)^{\alpha}}{\lambda\left(1+\beta\left(\frac{m}{i}\right)^{\alpha}\right)} \end{aligned} \tag{1}$$

Proof: see Appendix A.

where β is the SIR decoding threshold, b_i is a Bernoulli random variable with parameter p_i, node i transmits with probability p_i (the specified transmitter transmits with probability p_{tr}), r_i denotes the distance from the interferer i to the receiver (Fig. 2), λ is the fading exponential parameter, and i and m denote the index of interferer i and receiver respectively. Our assumption is that vehicles are located around the receiver to infinity symmetrically. If the channel access probabilities are constant and equal, the closed-form packet success probability is ($\alpha=2$):

$$\begin{aligned} P_s = &\frac{(1+\beta)[p+\lambda(1-p)]^2}{\lambda(1-p)[p+\lambda(1-p)+\lambda(1-p)\beta]}\cdot \\ &\frac{\left[\sinh \pi\sqrt{\frac{\lambda(1-p)\beta}{p+\lambda(1-p)}}m\right]^2}{\left(\sinh\sqrt{\beta}m\right)^2} \end{aligned} \tag{2}$$

Proof: see Appendix B.

If x denotes the distance between two adjacent nodes, mx represents the distance between receiver and transmitter. It is noteworthy to mention that equations 1 and 2 do not

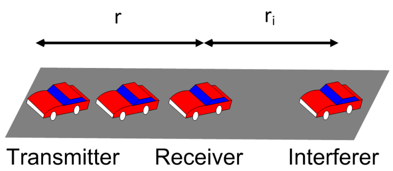

Fig. 2. A chain of vehicles. Distance between the transmitter and the desired receiver $= r$. Distance between interferer i and desired receiver $= r_i$.

depend on the inter-vehicle distance.

There are two approaches for an N-lane highway. The first approach is called the Single Lane Abstraction (SLA) model. In this model, all the traffic lanes are mapped into one lane with the aggregated traffic intensity. Using this model, equations 1 and 2 can still be employed to obtain packet success probability. SLA model can be used only if $d^2 \ll mx^2$ in which d shows the distance between two adjacent lanes (see Appendix C). If this condition is not satisfied, we cannot ignore d. Therefore, packet success probability can be obtained using:

$$P_s = \frac{\lambda(1+\beta)}{p_{tr} + \lambda(1-p_{tr}) + \lambda(1-p_{tr})\beta} \cdot \prod_{i \in -\infty - \{0\}}^{+\infty} \frac{p_i + \lambda(1-p_i) + \lambda(1-p_i)\beta \left(\frac{mx}{ix + \frac{d^2}{2ix}}\right)^2}{\lambda\left(1 + \beta\left(\frac{mx}{ix+\frac{d^2}{2ix}}\right)^2\right)} \tag{3}$$

If the time slots in which nodes transmit are not synchronized, this scheme is named *Slotted Asynchronous P-persistent* (SAP). In this case, an interferer can potentially interfere with *at most* two time slots of another transmission. Hence, the transmission probability for the interferers is:

$$p'_i = p_i + p_i - p_i \cdot p_i \simeq 2p_i \tag{4}$$

Since the probabilities are small, this approximation is tight.

2) Poisson Distribution:
Poisson point processes have been widely employed as a model for wireless networks [17], [18]. In this case, the nodes are distributed on a highway according to a Poisson point process. The packet success probability can be obtained by considering the fact that the transmitter-receiver distance is a random variable, not a constant value.

$$P_S = Prob(SIR > \beta) \tag{5}$$
$$= \int_r Prob\left(\frac{Phr^{-\alpha}}{k+I} > \beta\right) f_R(r)dr \tag{6}$$
$$= \int_r Prob\left(h > \frac{\beta(k+I)r^{\alpha}}{P}\right) f_R(r)dr \tag{7}$$
$$= \int_r e^{\frac{-\lambda\beta k r^{\alpha}}{P}} \cdot E_I\left[e^{\left(\frac{\lambda\beta r^{\alpha}}{P}\right)}\right] f_R(r)dr \tag{8}$$
$$= \int_r e^{\frac{-\lambda\beta k r^{\alpha}}{P}} \cdot L_I\left(\frac{\lambda\beta r^{\alpha}}{P}\right) f_R(r)dr \tag{9}$$
$$\text{if} \quad (k = 0, \alpha = 4, P = 1)$$
$$= \int_r L_I\left(\lambda\beta r^4\right) f_R(r)dr \tag{10}$$

where P, h, r, α, k, I, and λ represent the transmitter signal power, the channel fading coefficient, the distance between transmitter and receiver, the path loss exponent, the noise variance, the interference, and the exponential parameter of Rayleigh fading respectively. Assuming the transmitter and receiver are located on the same lane, the distribution of the distance between transmitter and receiver is Erlang.

$$f_R(r) = \frac{\lambda_p^n r^{n-1} e^{-\lambda_p r}}{(n-1)!} \tag{11}$$

in which λ_p represents the intensity of vehicles on a lane. Also, n denotes the number of nodes between transmitter and receiver plus one. [10] obtains closed-form expressions for the Laplace transform of the approximate aggregate interference. For this specific scenario, this Laplace transform is equal to:

$$L_I\left(\lambda_p\beta r^4\right) = e^{-\pi\lambda_M\left[b^2\left(1-e^{-\lambda_p\beta K\left(\frac{r}{b}\right)^4}\right) + \left(\lambda_p\beta r^4 K\right)^{\frac{1}{2}}\Gamma(0.5, 0.5Kb^{-4})\right]} \tag{12}$$

in which

$$\Gamma(s,x) = \int_x^{\infty} t^{s-1} \cdot e^{-t} dt \tag{13}$$
$$K = \left(\frac{c}{4\pi f_c}\right)^2 \tag{14}$$

Also, b, c, f_c represent the radius from the receiver node in which the aggregate interference is considered, the speed of radio propagation, and the carrier frequency. λ_M denotes the intensity of the simultaneously active nodes from the parent Poisson point process.

We employ a modified hard core point process type II (HCPP-II) model [10], [7] to find the intensity of the simultaneously active nodes. HCPP-II model mitigates the node intensity underestimation problem compared to HCPP model. In the modified HCPP-II model, we want to reduce the number of unselected nodes even more.

Let's assume $\Phi = \{x_i; i = 1, 2, 3, \cdots\}$ are the nodes in the network. The probability of having n points in a lane (lane length = L) is

$$Prob(|\Phi| = n) = \frac{(\lambda L)^n e^{-\lambda L}}{n!}$$

Now, we employ the concept of marked point processes since we want to include additional information about the points in the model. A marked point (n_i, m_i) is selected to be retained if and only if it has the lowest mark m_i in a circle of radius r centered at x_i (HCPP-II model). r denotes the minimum distance between any two simultaneously active transmitters. If we assume that the distribution of the marks in one circle is Uniform, then the probability of retaining a random point can be written as:

$$P_1 = \sum_{n=0}^{\infty} \frac{1}{n+1} Prob(\text{having n points in the lane}) \tag{15}$$

$$= \sum_{n=1}^{\infty} \frac{1}{n+1} \frac{(\lambda_p L)^n e^{-\lambda_p L}}{n!} \tag{16}$$

$$= \frac{1 - e^{-\lambda_p L}}{\lambda_p L} \tag{17}$$

Although this equation leads to a considerable improvement compared to the HCPP model, it has been shown that adding one condition to this model will enhance the system. Let's assume $D_{x_i}(r)$ represents the disk of radius r centered at x_i. The point x_i is retained in Φ_F if

a) $(D_{x_i}(r) \bigcap \Phi) \backslash x_i = \{x_j\}$ such that $m_{x_i} < m_{x_j}, \forall x_j \in (D_{x_i}(r) \bigcap \Phi) \backslash x_i$.
b) $(D_{x_i}(r) \bigcap \Phi) \backslash x_i = x_L \bigcup \{x_j\}$ such that $m_{x_i} > m_{x_L}$ and $m_{x_i} < m_{x_j}$, $\forall x_j \in (D_{x_i}(r) \bigcap \Phi) \backslash \{x_i, x_L\}$ given that $S(d) \bigcap \Phi = \{x_k\}$ such that $m_{x_L} > m_{x_k}$, $\exists k \in S(d) \bigcap \Phi$. In other words, the set $S(d) = D_{x_L}(r) \backslash D_{x_i}(r)$ contains at least one point with lower mark than x_L (Fig. 3).

To put it differently, the point $x_i \in \Phi$ is retained in Φ_F 1. if it has the lowest mark in $D_{x_i}(r)$, or 2. if it has a second lowest mark in $D_{x_i}(r)$ given that the point x_L with the lowest mark in $D_{x_i}(r)$ does not have the lowest mark in its own disc $D_{x_L}(r)$. In Fig. 3, according to the HCPP-II model, the point A with mark 0.7 is not retained because the point B with mark 0.6 exist in the point A's disk. However, in accordance with the modified HCPP, the point with mark 0.7 is retained since the point with mark 0.5 does not permit the point with mark 0.6 to be retained. Therefore, this model mitigates the node intensity underestimation problem of the traditional HCPP. Deriving the probability of the second part, (P_2), is similar to the derivation of P_1:

$$\begin{aligned} P_2 &= \sum_{n=1}^{\infty} \frac{1}{n+1} \frac{N^n e^{-N}}{n!} \sum_{k=1}^{\infty} \frac{k}{n+k+1} \frac{M^k e^{-M}}{k!} \\ &= \frac{M e^{-(N+M)}}{N} \\ &\cdot \left(\sum_{n=1}^{\infty} \frac{N^{n+1}}{(n+1)!} \sum_{k=1}^{\infty} \frac{1}{n+k+1} \frac{(M^{k-1})}{(k-1)!} \right) \\ &= \frac{M e^{-(N+M)}}{N} \\ &\cdot [\frac{e^{N+M} - 1}{N+M} + \frac{(M-N)(1-e^M) - NMe^M}{M^2}] \end{aligned}$$

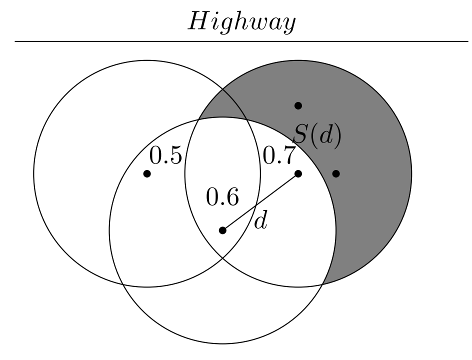

Fig. 3. Figure explains the modified HCPP. $S(d)$ is the set of points in the gray region.

where k is the number of nodes in $S(d)$, n denotes number of nodes in $D_{x_i}(r)$, N is the expected number of nodes in $D_{x_i}(r)$, and M is the expected number of nodes in $S(d)$. Let's consider two different cases:

a) Single Lane: In a single lane scenario (length L), $N = \lambda_p L$, $M = \lambda_p E(d)$. d denotes the distance between two nodes x_i and x_L. The distribution of d can be assumed to be the Erlang distribution with parameter λ_p.
b) General case: In this case, $N = \lambda_p \pi r^2$ and $M = \lambda_p E_d[S(d)]$ (E_d is the expectation over the random variable d). $S(d)$ is equal to $\pi r^2 - 2r^2 cos^{-1}(\frac{d}{2r}) + d\sqrt{r^2 - d^2/4}$. Also, the distribution of d is given by $f(d) = \frac{2d}{r^2}$, $0 < d < r$ [19].

Therefore, the probability of retaining a random point x_i is:

$$\begin{aligned} P_{tot} &= P_1 + P_2 \\ &= \frac{1 - e^{-N}}{N} + \frac{M e^{-(N+M)}}{N} \\ &\cdot [\frac{e^{N+M} - 1}{N+M} + \frac{(M-N)(1-e^M) - NMe^M}{M^2}] \end{aligned}$$

The intensity can be obtained as follows:

$$\lambda = P_{tot}/\lambda_p$$

TABLE I
IEEE 802.11P DATA RATES AND CORRESPONDING SIR DECODING THRESHOLDS

R (Mbps)	3	4.5	6	9	12	18	24
β (db)	5	6	8	11	15	20	25

IV. Numerical and Simulation Evaluation of Design

Communications delay is a main factor that influences the collision probability. It is sufficient that the message be received successfully only one time. As a result the successful reception at vehicle V_i in a chain of vehicles has a geometric distribution with parameter

$$P_s(i) \cdot p_{tr} \cdot (1 - p_i) \tag{18}$$

where $P_s(i)$ is the packet success probability. Also, p_{tr} and p_i represent the channel access probability for the transmitter and the desired receiver respectively. This parameter demonstrates the probability that the transmitter is sending messages, the desired receiver is obtaining the warnings, and the warning messages are successfully delivered, all simultaneously. This gives us the number of required slots on average for vehicle V_i to receive vehicle V_0's messages:

$$s(i) = \frac{1}{P_s(i) \cdot p_{tr} \cdot (1 - p_i)} \tag{19}$$

If SAP scheme is employed, we need to alter the equation:

$$s(i) = \frac{1}{P_s'(i) \cdot p_{tr} \cdot (1 - p_i')} \tag{20}$$

in which p_i' represents the channel access probability when the time slots are not synchronized and $P_s'(i)$ denotes packet success probability using the new channel access probabilities.

The allowable number of transmission opportunities within the tolerable delay period is:

$$D = \lfloor \frac{T(i)R}{L} \rfloor \tag{21}$$

R represents the data rate which is chosen from TABLE I while L denotes the packet length. $T(i)$ denotes the maximum tolerable delay to inform vehicle V_i which can be obtained from vehicles' equations of motion. Let P_s^D denote the success probability at V_j after D transmission opportunities:

$$P_s^D = 1 - (1 - s(j)^{-1})^D$$

This equation demonstrates the dependence of packet success probability on channel access probabilities and inter-vehicle distances (Fig. 4).

We compare the equal distance model, the HCPP model, and the modified HCPP model via MATLAB simulations in order to compare different estimates of the collision probability. Fig. 5 illustrates the vehicles' collision probability versus the number of vehicles. If we use the same simulation parameters for the equal-distance scenario, more collisions happen. This

TABLE II
SIMULATION PARAMETERS

Distribution	Poisson Equal distance
Velocity	$20\frac{m}{s}$
Deceleration rate	$[-4, -8]\frac{m}{s^2}$
Number of vehicles in an specific lane	8
Total number of vehicles	30
SIR decoding threshold	11 dB
Data rate	9 Mbps
Packet length	250 Bytes

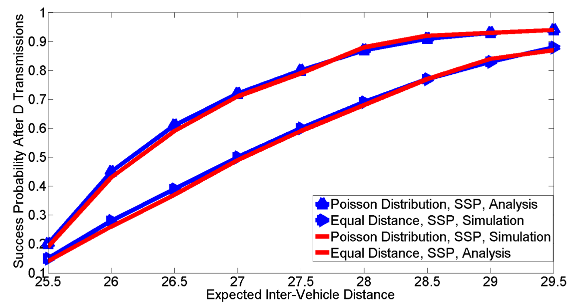

Fig. 4. Packet success probability after D transmissions at vehicle V_2 for different traffic models and different expected inter-vehicle distance. Parameters are given in TABLE. II

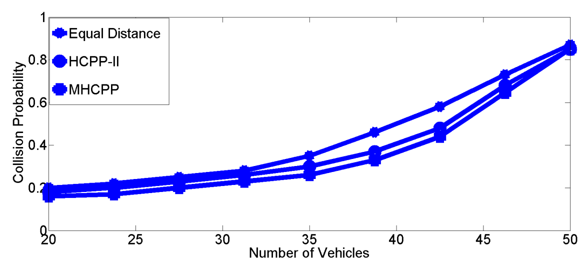

Fig. 5. Collision probability versus the number of vehicles for three models of vehicles in traffic. All the parameters are given in TABLE II except for the number of vehicles which is an independent variable.

seems to be justifiable because the equal-distance model represents the dense traffic.

The two extreme scenarios which we considered in this paper lead us to the conclusion that we may achieve a model-free approach to improve the performance of VANETs since both of the examined scenarios benefit the drivers following a similar pattern.

V. Conclusion

In this paper, we derived the equations of packet success probability for different traffic models in a network of vehicles. In addition, we compared both vehicle collision probabilities and packet success probabilities for different spatial distributions of vehicles. By taking these equations into account, the fatalities on highways will be reduced. Also, deriving these equations provide valuable insight into designing collision warning systems.

APPENDIX A
PROOF OF EQUATION 1

If there is distance r between a transmitter and the desired receiver, the success probability is

$$P_s = P\left(\frac{hr^{-\alpha}}{I} > \beta\right) \tag{22}$$

$$= \int P(h > \beta r^{\alpha} I | I = i) f_I(i) di \tag{23}$$

$$= E\left[e^{-\beta r^{\alpha} \sum_{i\in\Phi} b_i h_i r_i^{-\alpha}}\right] \tag{24}$$

$$= \prod_{i\in\Phi}\left[\frac{p_i}{\lambda\left(1+\beta r^{\alpha} r_i^{-\alpha}\right)} + (1-p_i)\right] \tag{25}$$

Assuming,

$$r = mx \quad \text{and} \quad r_i = ix \tag{26}$$

equation 1 is obtained.

APPENDIX B
PROOF OF EQUATION 2

Using Euler's product formula, we obtain the second equation.

$$sin(\pi z) \equiv \pi z \prod_{i=1}^{\infty}\left(1 - \frac{z^2}{i^2}\right) \tag{27}$$

APPENDIX C
DIFFERENCE BETWEEN SLA MODEL AND MULTI-LANE MODEL

Assume d is the distance between two specific lanes, x denotes the distance between two adjacent vehicles, and the transmitter is located on the middle lane. Let's assume r specifies the distance between transmitter and receiver (which is on a lane with distance d from the middle lane).

$$r = mx\sqrt{1+\left(\frac{d}{mx}\right)^2} \tag{28}$$

$$\approx mx\left(1+\frac{\left(\frac{d}{mx}\right)^2}{2}\right) \tag{29}$$

$$= mx + \frac{d^2}{2mx} \tag{30}$$

Therefore, if $d^2 \ll mx^2$, $r \approx mx$. Hence, we can assume the receiver is in the middle lane too. If not, that approximation cannot characterize the performance of vehicular networks on N-lane highways.

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